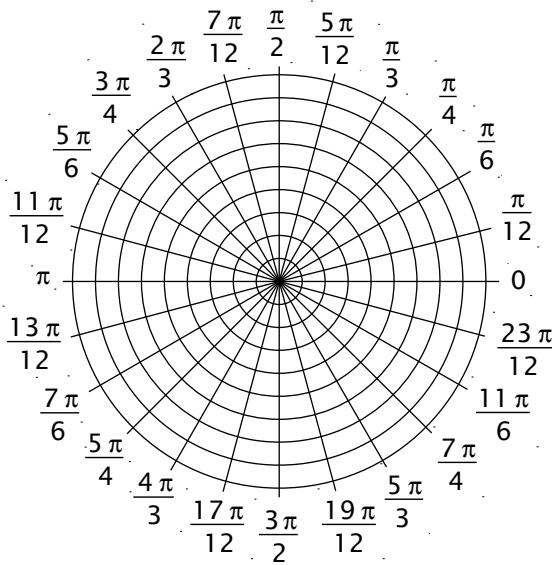# PreCalculus Placement Test

To receive the full benefit of this test, watch the student to ensure he has mastered the concepts presented in PreCalculus.
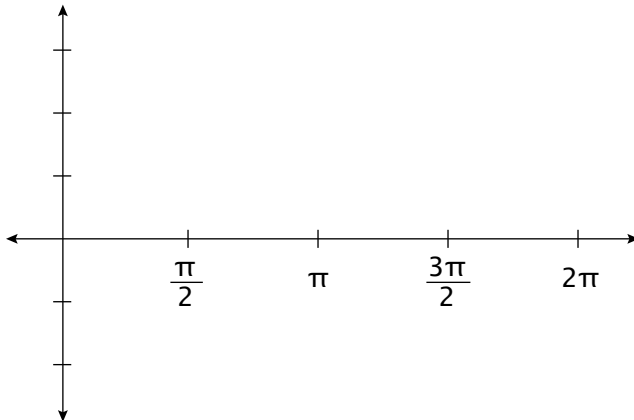If he demonstrates proficiency, he is ready to move on to Calculus.
If he struggles with the material on this exam, he should begin in PreCalculus.
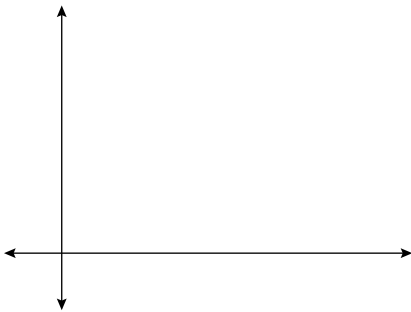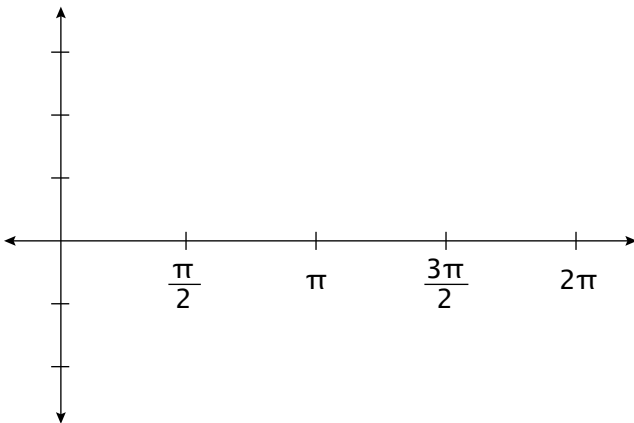
Graph.

1. $r = \dfrac{2}{\cos\theta}$

2. $y = 3 \sin x$

3. $f(x) = e^x$

4. $y = \tan x$

$\dfrac{\pi}{2}$ π $\dfrac{3\pi}{2}$ 2π

Solve for x.

5. $e^{2x} - e^x = 2$

6. $\dfrac{\sqrt{x+3}}{2} < 1$

7. $|x - 2| < -1$

8. $e^{2x} = 5$

Prove the identities.

9. $\tan\theta \csc\theta = \sec\theta$

10. $\csc^2\theta[\sin^2(90° - \theta)] + 1 = \csc^2\theta$

Evaluate.

11. $\lim\limits_{x \to \infty} \dfrac{1}{x}$

12. $\lim\limits_{x \to -5} \dfrac{x^2 + 3x - 10}{x + 5}$

13. $\sum\limits_{i=1}^{4} \{i^2 - 1\}$

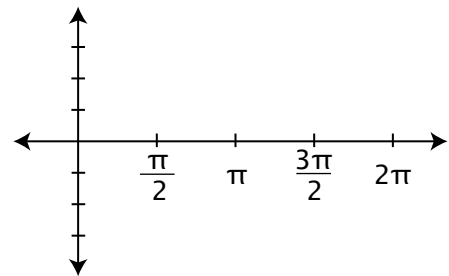14. $\sin 135° + \cos 60°$

Follow the directions.

15. If $f(x) = 2X - 3$ and $g(x) = X^2 + 1$, find $f(g(x))$.

16. Give the domain and range of the following function: $f(x) = \sqrt{x + 3}$
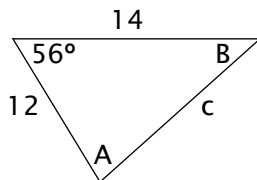
17. Change $7\pi/4$ radians to degrees.

18. Give the reference angle and the quadrant for 250°.

19. Graph 2 sin x – 1, using the graph at right.

20. Find the first four terms of the following geometric sequence, with $a_1 = -4$ and r = 1/2.
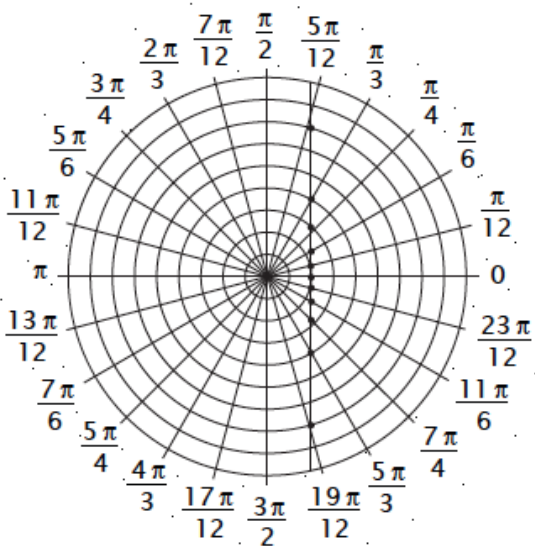
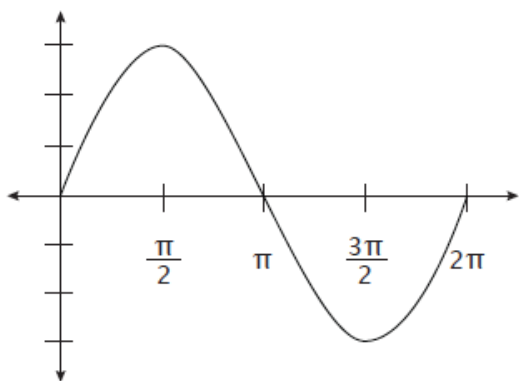21. Solve for the unknown sides and angles for the triangle shown.

22. The decay constant of a substance is determined to be .0069. How much of 10 grams will remain after 365 days? Use $Q(t) = 10e^{-kt}$, where t = time in days and Q(t) is the quantity remaining at time t.
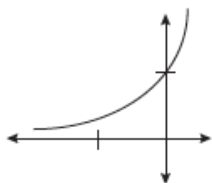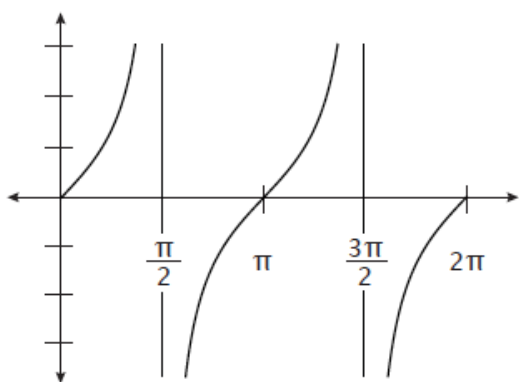
PreCalculus Solutions

1.

2.

3.

4.

5.
$$e^{2X} - e^{X} = 2$$
$$Y^2 - Y = 2 \ (\text{substituting Y for } e^{X})$$
$$Y^2 - Y - 2 = 0$$
$$(Y + 1)(Y - 2) = 0$$

$Y + 1 = 0$	$Y - 2 = 0$
$Y = -1$	$Y = 2$
$e^{X} = -1$	$e^{X} = 2$
$\ln e^{X} = \ln(-1)$	$\ln e^{X} = \ln 2$
not a real solution	$X = \ln 2$
	$X \approx .6931$

6.
$$\frac{\sqrt{X+3}}{2} < 1$$
$$\sqrt{X+3} < 2$$
$$\left(\sqrt{X+3}\right)^2 < 2^2$$
$$X + 3 < 4$$
$$X < 1$$

7. No real solutions because absolute value is always positive.

8.
$$e^{2X} = 5$$
$$\ln e^{2X} = \ln 5$$
$$2X \approx 1.6094$$
$$X \approx .8047$$

9.
$$\tan\theta \csc\theta = \sec\theta$$
$$\left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\sin\theta}\right) = \frac{1}{\cos\theta}$$
$$\frac{1}{\cos\theta} = \frac{1}{\cos\theta}$$

10.
$$\csc^2\theta\left[\sin^2(90°-\theta)\right] + 1 = \csc^2\theta$$
$$\frac{1}{\sin^2\theta}\left[\cos^2\theta\right] + 1 = \frac{1}{\sin^2\theta}$$
$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$
$$\frac{\cos^2\theta + \sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$
$$\frac{1}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

11.
$$\lim_{X \to \infty} \frac{1}{X} = 0$$
As the numerator grows larger, the fraction grows smaller, but will never reach or pass 0.

12.
$$\lim_{X \to -5} \frac{X^2 + 3X - 10}{X + 5} =$$
$$\lim_{X \to -5} \frac{(X+5)(X-2)}{X+5} =$$
$$\lim_{X \to -5} X - 2 =$$
$$\lim_{X \to -5} (-5) - 2 = -7$$

13. $\displaystyle\sum_{x=1}^{4} \{x^2 - 1\} = (1^2 - 1) + (2^2 - 1) + (3^2 - 1) + (4^2 - 1)$
$$= (1-1) + (4-1) + (9-1) + (16-1)$$
$$= 0 + 3 + 8 + 15$$
$$= 26$$

14. $\sin 135° + \cos 60° = -\sin 45° + \cos 60°$
$$= \frac{\sqrt{2}}{2} + \frac{1}{2}$$
$$= \frac{1 + \sqrt{2}}{2}$$

15. $f(g(x)) = f(x^2 + 1)$
$$= 2(x^2 + 1) - 3$$
$$= 2x^2 + 2 - 3$$
$$= 2x^2 - 1$$

16. The domain will be all X where $X + 3$ is greater than or equal to 0:
$$X + 3 \geq 0$$
$$X \geq -3$$
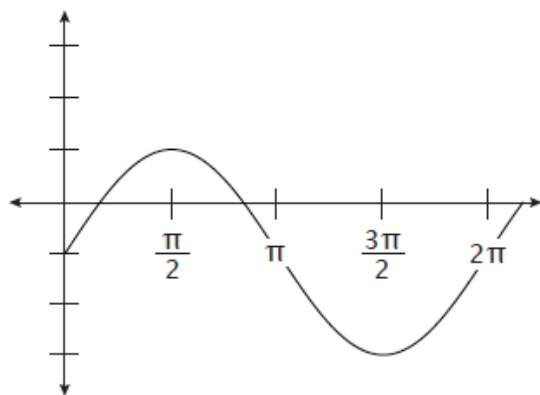domain: $[-3, \infty)$, or all numbers ≥ -3
range: $[0, +\infty)$

17. $\left(\dfrac{7\pi}{4}\right)\left(\dfrac{180°}{\pi}\right) = \dfrac{(7)(180°)}{4} = \dfrac{1,260°}{4} = 315°$

18. 70°; quadrant III

19.

20. $a_1 = -4$
$$a_2 = a_1 r = (-4)\left(\frac{1}{2}\right) = -2$$
$$a_3 = a_2 r = (-2)\left(\frac{1}{2}\right) = -1$$
$$a_4 = a_3 r = (-1)\left(\frac{1}{2}\right) = -\frac{1}{2}$$
$$-4, -2, -1, -\frac{1}{2}$$

21. $c^2 = a^2 + b^2 - 2ab\cos C$
$$c^2 = (14)^2 + (12)^2 - 2(14)(12)\cos 56°$$
$$c^2 \approx 196 + 144 - 336(.5592)$$
$$c^2 \approx 340 - 187.89$$
$$c^2 \approx 152.11$$
$$c \approx \sqrt{152.11}$$
$$c \approx 12.33$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{(14)}{\sin A} = \frac{(12.33)}{\sin(56°)}$$
$$(12.33)(\sin A) = (14)(\sin 56°)$$
$$\sin A = \frac{(14)(\sin 56°)}{12.33}$$
$$\sin A \approx \frac{(14)(.8290)}{12.33}$$
$$\sin A \approx \frac{11.606}{12.33}$$
$$\sin A \approx .9413$$
$$A \approx \arcsin .9413$$
$$A \approx 70.3°$$

$$B = 180° - (70.3° + 56°)$$
$$B = 180° - 126.3°$$
$$B = 53.7°$$

22. $Q(t) = 10e^{-kt}$
$$Q(365) = 10e^{-(.0069)(365)}$$
$$Q(365) = 10e^{-2.5185}$$
$$Q(365) \approx 10(.0806)$$
$$Q(365) \approx .806 \text{ g}$$